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The RASC method for ranking and scaling of biostratigraphic events

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Abstract

The RASC method for ranking and scaling of biostratigraphic events was developed by the authors and associates during the past 20 years. The purpose of ranking is to create an optimum sequence of events observed in different wells or sections subject to stratigraphic inconsistencies in the direction of the arrow of time. These inconsistencies, which result in crossovers of lines of correlation between sections, are due to various sampling errors and other sources of uncertainty including reworking and misclassification. They can be resolved by statistical averaging combined with stratigraphic reasoning. Subsequent scaling of the events can be carried out by estimating intervals between successive events along a relative time-scale. This results in the scaled optimum sequence. Either the ranked optimum sequence or the scaled optimum sequence can be used for biozonation and for correlation between sections with error bars to denote precision of the correlation. The observed positions in the sections of different biostratigraphic events can have different degrees of precision. These differences can be evaluated by analysis of variance. Other types of stratigraphic events including logmarkers can be integrated and incorporated with the biostratigraphic events. The purpose of this review paper is to provide a succinct description of RASC as it is today using the Cretaceous microfossil example of Gradstein et al. [Gradstein, F.M., Kaminski, M.A., Agterberg, F.P., 1999. Biostratigraphy and paleoceanography of the Cretaceous Seaway between Norway and Greenland. Earth-Science Reviews] for illustration.

Keywords: RASC method; ranking; scaling; biostratigraphic event

1. Introduction

RASC is an acronym for ranking and scaling of biostratigraphic events. There have been three distinct RASC method development phases:

(1) 1978–1985: Starting from an algorithm for ordering biostratigraphic events proposed by Hay (1972), the basic concepts of RASC were introduced and applied initially to Cenozoic Foraminifera in offshore wells drilled along the northwestern Atlantic Margin (Gradstein and Agterberg, 1982). Other applications and evaluations of the method include Doeven et al. (1982) and Harper (1984). Implementation of the method was in the form of mainframe computer programs (Agterberg and Nel, 1982a,b; Agterberg et al., 1985; Heller et al., 1985). Methodology as well as applications were presented in detail in Gradstein et al. (1985). Development of RASC was carried out at the Geological Survey of Canada

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in Ottawa and Dartmouth, Nova Scotia, as part of Project 148 of the International Geological Correlation Program (Evaluation and Development of Quantitative Stratigraphical Correlation Methods, 1976–1985).

(2) 1986–1994: Minor modifications of the method were introduced (Agterberg and Gradstein, 1988; D'Iorio and Agterberg, 1989; Agterberg, 1990). Implementation was in the form of programs for personal computers (Agterberg and Byron, 1990; Agterberg et al., 1989). New types of applications were described in Williamson (1987), Van Buggenum (1991), Gradstein et al. (1992), Schioeler and Wilson (1993), Agterberg and Gradstein (1994) and Gradstein et al. (1994).

(3) 1995–1998: Modifications of RASC with addition of variance analysis are introduced in Agterberg and Gradstein (1997a,b, 1999), Agterberg et al. (1998), and Gradstein and Agterberg (1998). Novel applications include Gradstein and Bäckström (1996), Whang and Zhou Di (1997), and Gil-Bescos et al. (1998). Microcomputer application is being facilitated by the development of a user-friendly Windows version of RASC to complement and replace the latest DOS version of Agterberg and Gradstein (1996). This last stage of development has been sponsored by Saga Petroleum. RASC has had other large-scale, practical applications by oil companies including Unocal, Shell, Arco, British Petroleum, Norsk Hydro, and British Gas.

The purpose of this paper is to provide a succinct description of RASC using the Cretaceous microfossil data of Gradstein et al. (1999) for illustration. This data set contains 1753 records on occurrences of 517 stratigraphic events in 31 wells.

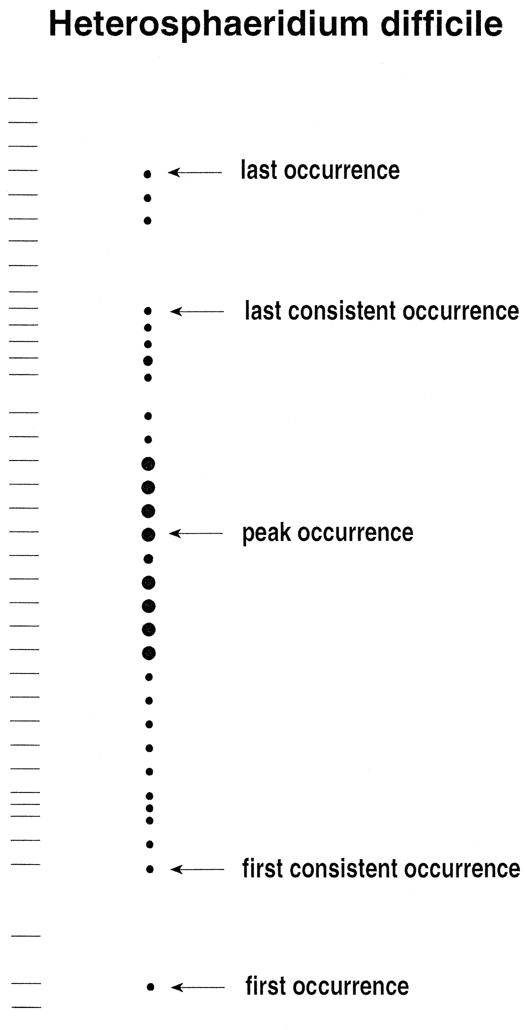

Fig. 1. In order to increase biostratigraphic resolution, attempts are made to recognize up to five correlative events along the stratigraphic range of a taxon, as shown here for the (theoretical) total range of *Heterosphaeridium difficile*. Unfortunately, such events often have limited geographical traceability. The vertical scale shows sampling levels in relative time (after Gradstein and Agterberg, 1998).

2. Data preparation and frequency distribution analysis

2.1. Fossil events—biostratigraphic events defined on fossils

As illustrated in Fig. 1, different types of stratigraphic events can be defined for the same fossil taxon. In practice, the occurrence of these events may be recorded directly, by means of samples taken at different stratigraphic locations (e.g., depths along a exploratory well, or distances measured in the stratigraphically upward direction from the base of an outcrop section). Indirect recording of events occurs when presences/absences of taxa are generalized over all samples, e.g., in a stratigraphic range chart, so that the events remain to be defined for each stratigraphic section (cf. Fig. 1). Commonly, the presence of a taxon in a sample is qualified by provision of a measure of its abundance.

The basic input for RASC consists of a list of event names (dictionary) and a well data file consisting of multiple records of the events in all wells considered. Many RASC applications are based on data derived from exploratory wells, as in the example of this paper. In each well, the events observed to occur in the samples are characterized by their depths. However, it should be kept in mind that the method can be applied equally well to stratigraphic events observed in land-based outcrop sections.

2.2. Frequency distributions—to analyze presences and absences of fossil events

The most important feature of RASC is probably its ability to cope with missing data. Ideally, every stratigraphic event selected would occur in all wells or sections considered. However, particularly in the situation of well-sampling, the frequency distribution of the events is positively skewed. This means that most events are observed in one or a few sections only, whereas no or relatively few events are observed in all or most sections. There are many reasons why the occurrence of a fossil taxon is rare. These include small probability of preservation, small size of well-samples, and provincialism (occurrence of taxa in restricted domains).

For the example of Cretaceous microfossils, this skewness is illustrated in fig. 4 of Gradstein et al. (1999). When the vertical scale is arithmetic, the curve for the cumulative frequency distribution is ‘hollow’ with rapidly decreasing slope along the horizontal axis. For the purpose of applying RASC, it is not necessary to model the frequency curve statistically. However, this kind of frequency curve usually can be straightened out and loses its hollowness when the vertical scale is made logarithmic as in Fig. 2. This indicates that the underlying type of statistical frequency distribution is approximately exponential.

Ranking and scaling are performed on observed superpositional relationships between events co-occurring in the same sections. The frequency distribution of all possible pairs of co-occurring events is even more skewed than the frequency distribution of single events (Fig. 2). In a typical example of application, there may be as many as 500 single observed events implying that there are $n(n-1)/2 = 124{,}500$

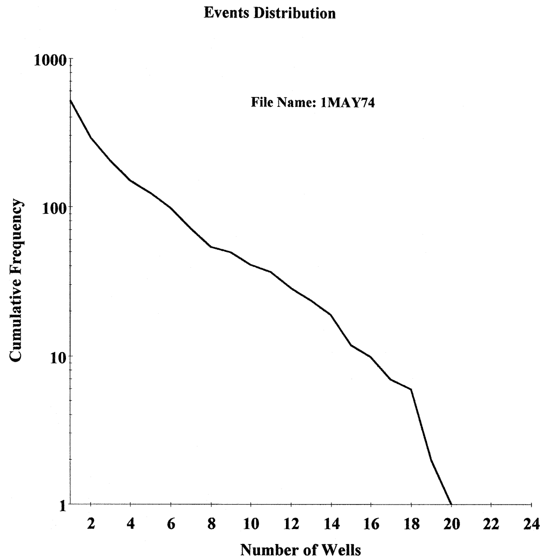

Fig. 2. Cumulative frequency distribution of 587 Cretaceous microfossil event occurrences in 31 wells, offshore Norway; same data as used for Fig. 4 in Gradstein et al. (1999) except that logarithmic scale is used in the vertical direction. From two to 18 wells the curve is approximately straight indicating that the underlying frequency distribution is exponential.

pairs of events. Many of these pairs are not, or hardly, represented in the data base and very few, if any, occur in most sections.

At the beginning of a RASC application, a threshold parameter has to be set for the minimum number of wells in which each event should occur. In general, this cut-off greatly reduces the total number of events considered and it has even more effect on the total number of pairs of events. Suppose that 50 of the 500 events in the preceding example are retained for the (scaled) optimum sequence. Then the number of possible pairs is reduced to 1225. Consequently, a 90% reduction in events corresponds to a reduction of over 99% in pairs of events.

2.3. Unique event option—to reinsert rare (index fossil) events into the (scaled) optimum sequence

It is necessary to set the threshold parameter for minimum sample size to facilitate application of the statistical techniques of ranking and scaling. There is, however, the drawback of loss of important information on index fossils that occur in one or only a few sections within the study area. For this reason, we have added a unique (rare) event option to RASC. A limited number of unique events (commonly fewer than 20) can be selected for insertion into the optimum sequence or scaled optimum sequence, after completion of the ranking or scaling calculations.

This insertion is based on relative positions in the (scaled) optimum sequence of the events occurring in the same samples and in samples adjacent to the samples containing the unique event. In general, the positions of the unique events in the (scaled) optimum sequence are less precise than those of the other events which occur more frequently. For this reason, the positions of unique events are clearly marked in the final results.

In the current version of RASC, all observed pairs of co-occurring events are used for ranking, even those co-occurring in a single section only. For scaling, however, it is desirable to set a second threshold parameter for minimum number of wells in which a pair of events should occur. This is because scaling is based on relative frequencies of different types of superpositional relationships for the same pairs of co-occurring events. Although the use of this second threshold increases the number of missing pairs of events, it improves the precision of interevent distances estimated from the relative frequencies based on pairs of events retained after the setting of the second threshold parameter. In general, the second threshold parameter should be one or more less than the first one. Otherwise, too few pairs of events would be retained for statistical analysis.

It is not always desirable to perform scaling. Especially for preliminary RASC runs on unedited data sets, the analysis may be restricted to ranking. Ranking followed by scaling was employed in the two types of RASC runs applied to the Cretaceous microfossil example: the two sets of threshold parameters used were 6/3 and 7/4, respectively. Table 1 shows the frequencies of unique events used in the 7/4 run.

Table 1
Frequencies of nine unique events used for 7/4 run on 587 Cretaceous microfossil event occurrences in 31 wells, offshore Norway (cf. Gradstein et al., 1999)

Number	Frequency	Name
20	4	*Praeglobotruncana stephani*
22	6	*Dicarinella imbricata*
23	5	*D. hagni*
38	5	*Ammoanita globorotaliaeformis*
342	3	*Pseudotextulariella cretosa*
353	5	*Apteodinium grande* LCO
371	6	*Fromea* sp. 2 Strl.
447	2	*Textularia* sp. 1 B and R81
526	5	*Sigmoilina antiqua*

These rare events are important for biozonation but will not be used during ranking and scaling calculations for the example in this paper because the (scaled) optimum sequence is constructed for events each occurring in at least seven wells.

3. Ranking—calculation of most likely sequence of fossil events in relative time

3.1. Modified Hay method—to rank events according to their superpositional relationships

The concept of an optimum sequence based on ranking was originally introduced by Hay (1972). All relative frequencies by which events are observed to occur stratigraphically above or below other events are considered. In the original Hay-type optimum sequence, the events are ranked in such a way that those above others in the optimum sequence have higher frequencies of occurring above these others, and lower frequencies of occurring below them.

This ranking is based on superpositional relationships between events. In general, there are three possible types of superpositional relationships for a pair of events co-occurring in the same section. An event can be observed to occur above or below another event, or the two events coexist in the same sample. This third type of relationship, which implies that the two events are observed to be coeval, can be ignored when an original Hay-type optimum sequence is constructed. In the ranked optimum sequence, which is based on a larger number of sections, two events can be coeval on the average when one of them occurs exactly as many times above the other one as it occurs below it.

If an event is observed above another event in some sections but below it in others, a stratigraphic inconsistency involving these two events is indicated. The purpose of constructing the optimum sequence is to resolve such inconsistencies. In fact, there is no point in applying the methods described in this paper if inconsistencies are missing when sections are compared with one another. Lines of correlation connecting observed positions of events in sections show crossovers when there are inconsistencies.

The so-called crossover frequency is defined as follows. If two events (A and B) co-occur in n sections with A above B in $n(\mathrm{AB})$ sections, A below B in $n(\mathrm{BA})$ sections, and A with B in the same sample in $n(\mathrm{A}-\mathrm{B})$ sections, then the crossover frequency of A with respect to B is $f(\mathrm{AB}) = \{n(\mathrm{AB}) + 0.5n(\mathrm{A}-\mathrm{B})\}/n$. Two events that are coeval on the average have crossover frequency equal to 0.5. When an event occurs above another event in all sections, the crossover frequency is one, indicating absence of inconsistency in the observed superpositional relationships.

3.2. Events cycles—inconsistencies involving three or more fossil events

In most applications of ranking to large data sets, it is not possible to construct an optimum sequence according to the criteria outlined at the beginning of this section. Pairwise inconsistencies, in which one event occurs above another in some sections but below it in other sections, are not the only possible type of inconsistency in superpositional relationships between events. Consider the following possibility: an event labeled A occurs more frequently above event B than the other way around, B occurs more frequently above C, and C above A. This is an inconsistency involving three events.

When a three-event cycle of this type occurs in the data set, it is not possible to construct the original Hay-type optimum sequence. However, by means of searching with the help of a computer, it is possible to identify those groups of three events which are involved in three-event cycles, and to study their superpositional relationships in detail. In RASC such three-event cycles, and cycles involving more than three events (see next paragraph), are identified one after another. When a three-event cycle is found, the superpositional relationship of the two events that have the smallest difference between frequencies of occurring above and below one other is ignored. Successive application of this rule to all three-event cycles encountered eventually results in a Hay-type optimum sequence.

In reality, the situation may be more complex. There may be many three-event cycles in a data set, but these can be identified and ‘broken’ one after another. However, more than three events may be involved in a single cycle. For example, if event A occurs more frequently above B, B above C, C above D, and D above A, the result is a four-event cycle, provided that A and C as well as B and D are coeval on the average. If the latter condition is not satisfied, the four-event cycle is not real, because then the four events considered contain at least one three-event cycle which would be identified as such.

Similar considerations apply to cycles involving more than four events. In general, the frequency of occurrence of four-event cycles is much less than that of three-event cycles and cycles with more than four events are even rarer. The first three cycles successively encountered in the 7/4 run of the Cretaceous microfossils are shown in Table 2. This method of ranking is called the modified Hay method.

3.3. Presorting—to rank events by scores assigned to frequencies of superpositional relationships

The modified Hay method is not the only method that can be used for ranking stratigraphic events. Other methods can be based on addition of relative frequencies of superpositional relationships involving successive events (Blank and Ellis, 1982), or on transformations of crossover frequencies. Before the modified Hay method is implemented in RASC, the following ranking method (called presorting) is applied first. Every event is compared with all other events. If an event occurs more frequently above another event, it receives an (additional) score of +1; if it occurs more frequently below the other event its score is not increased; if it is coeval on the average with another event, both event scores are increased by 0.5. The events can be ranked according to the magnitudes of their total scores in order to obtain an optimum sequence.

Ranking methods based on scores have drawbacks because of the following two factors: (a) information on superpositional relationships may be missing for

Table 2
First three of 85 three-event cycles (or four-event cycles) encountered during application of the modified Hay method for ranking

Cycling events:	34	29	277
Matrix elements:	0.0	2.5	3.5
	3.5	0.0	3.0
	1.5	9.0	0.0
(34, 29) zeroed—based on least difference between pairs			
Cycling events:	453	37	184
Matrix elements:	0.0	0.0	1.0
	4.0	0.0	0.0
	0.0	1.0	0.0
(37, 184) zeroed—based on least difference between pairs			
Cycling events:	137	184	453
Matrix elements:	0.0	0.0	2.0
	1.0	0.0	0.0
	0.0	1.0	0.0
(137, 184) zeroed—based on least difference between pairs			

The elements in the upper triangle of each matrix represent the frequency that the row component occurs above the column component, and the lower triangle is the reverse. The cycles are broken by temporarily zeroing the pair of events in the cycle with the smallest difference between their frequencies. If the frequencies in two or more pairs are equal to one another, only those of the first pair are ignored. All zeroed pairs are stored in a separate array so that they can be reinserted before scaling is applied.

many pairs of events; and (b) events near the top of (or higher up in) a sequence automatically always receive scores of $+1$ when compared with events near the base of (or lower down in) the sequence. Both factors can be a source of imbalance in the total score. In RASC the first factor (a) is counterbalanced by multiplying the total score of an event by the ratio between total number of other events with which it co-occurs in one or more sections and total number of other events. An imbalance created by the second factor (b) can not be readily corrected. For this reason, the modified Hay method, which is not subject to this type of imbalance, generally provides better results. This is because interchanges involving pairs of events generally are independent of the superpositional relations of these events with those near the top and base of the sections. The advantage of presorting is that, with little computational effort, it results in a preliminary ranking which is likely to provide a good, preliminary solution. In RASC, the presorting result is refined by means of the modified Hay method.

3.4. Choice of top and base of optimum sequence

The top and base of the stratigraphic sections should be carefully selected before ranking and scaling are applied, in order to avoid possible edge effects as much as possible. For example, it may be good to set the top or base at a major break in the stratigraphic record such as the K/T boundary. Otherwise, it is possible that some events are not correctly defined for sections because the fossils on which they are defined either already existed before or after the time interval used for the (scaled) optimum sequence.

Some problems related to choice of top of optimum sequence can be illustrated by means of a well known example previously statistically analyzed by Strauss and Sadler (1989) and Marshall (1995). Strauss and Sadler (1989, see their Fig. 1) constructed average values as well as 95% confidence limits for the endpoints of late Cretaceous ammonite ranges, Seymour Island, Antarctic Peninsula (original data from Macellari, 1986). The ammonites were numbered 0, 1, 2, . . . , 12.The beginning of an ‘optimum’ sequence for last occurrences based on Strauss and Sadler’s 95% confidence limits, in the stratigraphically downward direction, would be 11, 0, 6, 12, (5, 3, 7), 9. The parentheses in this sequence indicate that 5, 3 and 7 have approximately the same position (1230 m from the base of the section).

The taxa labeled 0, 3, 5, 6, and 12 disappear from the record at approximately the same level (1160 m). A RASC-type ranking solution for this example would be (0, 3, 5, 6, 12), 7, 9, 8, . . . , 4. Later an iridium layer was discovered just above the positions of occurrence of the last ammonites and Marshall (1995) assumes that this marks the K/T boundary which would provide a true cut-off point for all ammonite taxa considered. Thus endpoints above 1160 m would not be possible. If Marshall is right, the RASC solution would be more realistic than Strauss and Sadler’s solution near the top of the section. However, farther down in the section an optimum sequence based on the Strauss–Sadler solution may be better because these authors base their model on all observed occurrences of ammonites (instead of last occurrences only). Usually this type of detailed stratigraphic information is not available, especially not for exploratory well data.

3.5. Optimum sequence— end product of ranking

Fig. 3 shows the central part of the RASC optimum sequence of our Cretaceous microfossil example (*Palaeoglenodinium cretaceum* to *Textularia foeda*). The unique events of Table 1 were reinserted into the optimum sequence obtained by presorting followed by the modified Hay method. The sequence positions in the first column are followed by uncertainty interval (UI) numbers. These UI numbers are missing when an event has clearly defined position in the optimum sequence, viz. the event immediately above it occurs more frequently above it in all sections containing both events; and the same applies to the event immediately below it. However, two events can be coeval on the average. Then their successive UI numbers are (0 1) and (−1 0), respectively, indicating that the position number (first column) of the stratigraphically higher event could be increased by 1 with the second event taking its position in the optimum sequence.

More complex situations can arise in practice; for example, three or more events can be coeval on the average. This results in wider uncertainty intervals.

Ranked Optimum Sequence

No.	& UI			N	SD
1		368	Palaeoglenodinium cretaceum	14	1.492
2		370	Fenestrella belli	13	1.395
3		280	Rzehakina epigona group	9	2.581
4	0 1	194	Spherical/Lenticular radiolarians	15	1.594
5	-1 0	326	Haplophragmoides aff. walteri K&K	9	3.270
6		330	Inoceramus needles LCO	16	1.171
7	0 1	195	Stensioeina granulata polonica	14	1.069
8	-1 0	65	Nuttallinella florealis	7	3.076
9	0 1	369	Chatangiella sp. 1 Strl.	13	1.404
10	-2 0	503	Trithyrodinium 'reticulatum'	7	1.174
11		238	Surculosphaeridium longifurcatum	16	2.433
12		205	Heterosphaeridium difficile	14	1.028
13		82	Heterosphaeridium difficile LCO	20	1.408
14	0 2	445	Bulbobaculites problematicus	7	3.874
15	-2 0	63	Gaudryina filiformis	11	2.283
16		22	**Dicarinella imbricata		
17		209	Maghrebinia membraniphorum	10	3.129
18		23	**Dicarinella hagni		
19		502	Nyktericysta/Australosphaera	9	2.161
20		272	Stephodinium coronatum	18	1.912
21		16	Hedbergella delrioensis	17	1.865
22		20	**Praeglobotruncana stephani		
23		468	C/T boundary gamma spike	11	2.115
24		218	Endoceratium dettmanniae	14	2.139
25		13	Hedbergella planispira	15	2.441
26		84	Batioladinium jaegeri	10	2.301
27		73	Glomospirella gaultina	9	3.153
28		18	Whiteinella brittonensis	8	2.981
29		371	**Fromea sp. 2 Strl.		
30		212	Hedbergella delrioensis LCO	14	1.230
31		53	Litosphaeridium siphoniphorum	19	2.135
32		89	Epelidosphaeridia spinosa	13	2.627
33		219	Hedbergella planispira LCO	7	1.528
34		92	Rhombodella paucispina	9	1.656
35		447	**Textularia sp. 1 B&R81		
36		66	Valvulineria gracillima	11	2.305
37		75	Gavelinella intermedia	11	1.785
38		299	Xiphophoridium alatum	9	3.220
39		57	Textularia foeda	9	2.845

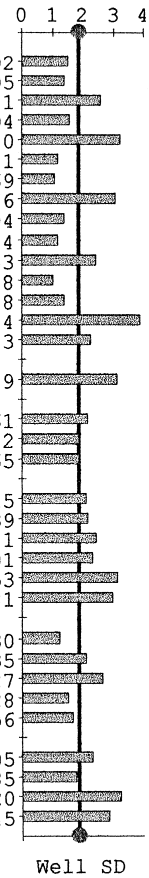

Fig. 3. Ranked optimum sequence for central part (Cenomanian–Santonian) of Cretaceous microfossil event example. UI: uncertainty interval (see text for explanation); *N*: sample size (number of wells in which event occurs); SD: standard deviation, also plotted along horizontal scale on the right; unique events are marked by double asterisks; Well SD: 'average' well standard deviation (see text for further explanation). Arrow of time points upwards.

If two events that are closely together in the optimum sequence do not co-occur in any well, this also would result in a wider UI. In general, many relatively large UI numbers indicate that the ranked optimum sequence is relatively poor. Then it may be possible to improve the quality of the solution simply by increasing the threshold which is the minimum number of wells in which each event should occur for inclusion in the optimum sequence.

The last part of Fig. 3 shows number (N) of wells in which an event occurs, and the event's standard deviation, which is also displayed graphically. This standard deviation is computed from deviations for each event from lines of correlation (see Section 5) constructed for all sections containing the event. If an event has relatively small standard deviation, it is close to all lines of correlation considered. This means that it is a relatively good marker. An 'average' standard deviation (Well SD) computed from all deviations in all wells is shown for comparison. These concepts will be explained in more detail later in Section 5.

3.6. Step model— to penalize events out of place in a stratigraphic section

The step model is the first of several methods applied in RASC to compare individual sections to the optimum sequence which can be regarded as an average or 'standard' based on all sections. The step model also can be applied after construction of a scaled optimum sequence.

The order of the stratigraphic events in a section that belong to those included in the optimum sequence is compared to their order in the optimum sequence in the following way. Each event is compared to all others. One penalty point is assigned when the order of two events in the section is not the same as in the order of these two events in the optimum sequence. Half a penalty point is assigned when two events are observed to be coeval. Consequently, an event that, in comparison with its neighbors, occurs much higher in a section than in the optimum sequence, e.g., due to reworking, will receive a relatively large number of penalty points, one for every event with which it is out of step in comparison to the optimum sequence. Similar considerations apply to an event that occurs stratigraphically too low in a section. In RASC output, the table

Table 3
Partial step model output for example of Fig. 3

Name (optimum sequence)	Well number						
	1	2	3	4	5	6	7
P. cretaceum			0.0				1.0
Fenestrella belli	2.0		0.0	3.0			1.5
Rzehakina epigona group					3.0		
Spherical/Lenticular radiolarians				3.0			
Haplophragmoides aff. *walteri* K and K				4.0	3.0		
Inoceramus needles LCO	1.0		0.0			0.0	1.0
Stensioeina granulata polonica	2.0						
Nuttallinella florealis	7.0			4.0			
Chatangiella sp. 1 Strl.						0.5	1.0
Trithyrodinium 'reticulatum'						0.5	1.0
Surculosphaeridium longifurcatum	2.0			2.0		0.0	3.0
H. difficile	2.0		0.0			0.0	0.0
H. difficile LCO	1.0	0.0		1.0		0.0	1.0
Bulbobaculites problematicus							
Gaudryina filiformis				1.0	0.0		1.0
Maghrebinia membraniphorum	1.0	1.0		7.0			7.5
Nyktericysta/ Australosphaera						0.5	2.0
Stephodinium coronatum	1.0	1.5				0.5	4.0
Hedbergella delrioensis	3.0			3.5			1.0
C/T boundary gamma spike				3.0			1.0
Endoceratium dettmanniae		4.0				0.0	2.5
H. planispira		6.0		2.5			
Batioladinium jaegeri		7.0		4.0		0.0	2.0
Glomospirella gaultina						2.0	
Whiteinella brittonensis	3.0			1.0			
H. delrioensis LCO		7.0				1.5	
Litosphaeridium siphoniphorum	3.0	6.0		2.0			1.5
Epelidosphaeridia spinosa	3.5	6.0		2.0			3.0
H. planispira LCO							
Rhombodella paucispina		5.0				1.5	1.5
Valvulineria gracillima	4.0	4.0					
Gavelinella intermedia							
Xiphophoridium alatum	7.0	10.0					1.5
T. foeda	5.0	8.5					

When the order of two events in a well is the reverse of their order in the optimum sequence, one penalty point is assigned. Events observed to co-occur in the same sample automatically receive 0.5 penalty point. If an event in the optimum sequence does not occur in a well, its space is left blank in the table. Separate tables (not shown here) can be consulted to check if events (with high penalty scores) that are out of place in a well occur stratigraphically higher or lower than expected. The first 34 rows and 7 columns of the complete (72×31) matrix are shown only.

of total number of penalty points for every event in every section including half-points for coeval events (see Table 3 for example) is followed by separate tables (not shown here) for events observed to occur stratigraphically too high and too low in comparison with the optimum sequence.

The penalty events scored for all events in a section can be added. The total number of penalty points of a section reflects how well the order of the events in the section resembles the optimum sequence but it also depends on the number of optimum sequence events occurring in the section. This total can be normalized to make it independent of total numbers of events in section and optimum sequence. Further standardization of the total number of penalty points per section yields Kendall's rank correlation coefficient or Kendall's 'τ' (Kendall, 1975) which, like other types of correlation coefficients, is restricted to the interval between 1 and -1. Kendall's τ equals one when the order of events in a section is exactly equal to that in the optimum sequence. This statistic provides a useful measure of how well each section corresponds to the optimum sequence (see Table 4 for example).

Table 4
Addition of all penalty scores for a well and standardization results in Kendall's rank correlation coefficient which provides a measure for the degree of correlation between the observed order of events in a well and their order in the (scaled) optimum sequence

Well No.	Kendall's τ
1	0.830
2	0.371
3	0.846
4	0.812
5	0.816
6	0.933
7	0.899
8	0.928
9	0.901
10	0.870
11	0.733
12	0.852
13	0.828
14	0.836
15	0.887
16	0.903
17	0.917
18	0.787
19	0.803
20	0.919
21	0.884
22	0.909
23	0.789
24	0.858
25	0.905
26	0.879
27	0.781
28	0.767
29	0.874
30	0.855
31	0.800

In the example, Well 2 has relatively small τ-value as also suggested by the (partial) set of penalty scores shown in Table 3 (7 of 13 values shown are 6 or >6).

4. Scaling—calculation of intervals between successive fossil events in relative time

A drawback of the ranked optimum sequence is that little is known about possible clustering of the events along the relative geological time scale. The uncertainty interval (UI in Fig. 3) only provides information about equally spaced positions of events within the optimum sequence. The purpose of scaling is to compute interevent distances which are intervals between successive events in the optimum sequence. It is logical that two events that are coeval on the average will be assigned zero interevent distance along this scale.

Scaling of the optimum sequence makes use of the stratigraphic reasoning that pairs of events in the optimum sequence with higher crossover frequency are closer together in relative time. Lines of correlation connecting observed positions of events in sections show crossovers when there are inconsistencies in the relative order of the events from section to section. Interevent distances are computed from z-values which, in turn, are calculated from the crossover frequencies by the 'probit' transformation $z_{ij} = \Phi^{-1}(f_{ij})$ where i and j are indexes representing two stratigraphic events; and Φ represents the cumulative frequency of the normal (Gaussian) distribution in standard form. The probit transformation is widely applied in biometrics (Finney, 1971), although not commonly for the purpose of scaling. It can be approximated by the linear transformation $z_{ij}^{*} = 2.93(f_{ij} - 0.5)$. For example, $f_{ij} = 0.5$ yields

$z_{ij} = z^*_{ij} = 0$; $f_{ij} = 0.9$ gives $z_{ij} = 1.28$ and $z^*_{ij} = 1.17$.

Every estimated interevent distance is the weighted average of a single direct distance estimate and $(n^* - 1)$ indirect distance estimates. The direct distance estimate is simply the z_{ij}-value of the two events considered. Usually, there are many more indirect distance estimates which are based on differences between z-values. Each difference $(z_{ik} - z_{jk})$ involves a third event with index k that occurs in the vicinity of i and j in the optimum sequence. An advantage of using a single linear scale is that, on the average, any difference involving a third event is equal to the direct estimate. The expected value (mathematical expectation) of a direct estimate and all indirect estimates of an interevent distance is the same. Discrepancies observed in practice between direct and indirect estimates can be ascribed to small-sample fluctuations and their effects reduced by averaging. The final estimated interevent distance is a function of the distance between each of the two events and all other events in the sequence which are in the vicinity of the two events.

Weighting is applied during the computation of the average of the n^* individual estimates in order to account for the fact that crossover frequencies based on relatively many superpositional relationships are more precise than those based on relatively few; during this weighting it also is considered that, taken separately, indirect estimates of distance are less precise than direct estimates. Standard deviations of the final interevent distances can be computed as well (see Agterberg, 1990, for mathematical equations).

The ranked optimum sequence is the starting point for all calculations outlined in the preceding two paragraphs. Usually, several of the computed interevent distances turn out to be negative. However, by adding successive interevent distances, it is possible to calculate a cumulative RASC distance for every event. This cumulative distance is measured from the first event in the optimum sequence which can be defined as origin. Both origin and scale of the axis along which the events are plotted can be changed arbitrarily, because this scale is relative. Using cumulative distances, every event obtains a fixed position along the scale and, consequently, all interevent distances between successive events become positive, even if, initially, one or more negative values had been obtained. This procedure is called reordering. If there are no negative values, the order of the events remains unchanged.

If negative interevent distances are computed, the preceding scaling calculations are repeated using the order of events in the initial scaled optimum sequence as the input sequence instead of the original ranking solution. The entire scaling process is repeated up to five times. In general, this iterative process results in elimination or significant reduction of the appearance of negative interevent distances during the calculations before final reordering is applied on the basis of the cumulative RASC distances. The question may be asked of why reordering is stopped after five iterations. The answer is that it can be shown theoretically that the preceding iterative reordering process does not necessarily converge toward a single stable solution and may continue to fluctuate between similar but slightly different solutions (Agterberg, 1990).

4.1. Dendrogram—to graphically represent the scaled optimum sequence

Table 5 shows part of the final scaling solution for the 7/4 run used for example. Note that all estimated interevent distances in this table are positive indicating that final reordering converged to a single stable solution; the distances shown are based on relatively large samples. Only the larger interevent distances significantly exceed their standard deviations.

As for the ranked optimum sequence of Fig. 3, the unique events can be included afterwards. The final product is shown graphically in the form of a dendrogram in Fig. 4. In this graph the interevent distances are plotted along a horizontal scale. At the end of each line segment, a vertical line is drawn in the stratigraphically downward direction until it hits another line (or the base line). Groups of events with short interevent distances form clusters which can be named and used for zonation (see Fig. 5 in Gradstein et al., 1999). Although the unique events are shown in the dendrogram, interevent distances involving them are not plotted, because they are less precise than the interevent distances calculated by scaling (Table 5).

Table 5
Statistics for partial final scaled optimum sequence (7/4 run) of Fig. 4

Event pairs	Interevent distance	Cumulative distance	Sample size	Weight	Standard deviation
368–280	0.0792	4.7542	14	20.0	0.1852
280–370	0.0989	4.8530	14	18.7	0.1678
370–194	0.1263	4.9793	15	19.4	0.1535
194–326	0.2220	5.2013	17	24.8	0.2092
326–65	0.0251	5.2264	17	17.6	0.1778
65–330	0.1995	5.4259	11	14.9	0.3137
330–195	0.2467	5.6726	16	18.2	0.1064
195–503	0.4012	6.0738	12	10.9	0.2047
503–369	0.2411	6.3149	13	14.9	0.0870
369–238	0.1028	6.4177	23	31.0	0.0739
238–205	0.0488	6.4665	22	29.4	0.1195
205–82	0.5460	7.0125	18	25.2	0.1337
82–63	0.4289	7.4413	28	31.1	0.0981
63–272	0.2410	7.6824	28	33.6	0.1320
272–502	0.1456	7.8280	11	23.5	0.1519
502–445	0.0106	7.8385	33	30.5	0.1012
445–209	0.0466	7.8851	28	28.1	0.0851
209–468	0.0341	7.9192	19	25.4	0.1338
468–16	0.0262	7.9455	28	34.5	0.0754
16–18	0.0516	7.9971	32	28.8	0.1070
18–218	0.1305	8.1276	29	25.7	0.1323
218–73	0.0951	8.2227	26	32.4	0.1279
73–13	0.0769	8.2996	28	36.1	0.1015
13–84	0.0558	8.3555	28	36.9	0.0947
84–212	0.4526	8.8081	20	31.5	0.0986
212–53	0.0481	8.8562	28	44.7	0.0707
53–89	0.2160	9.0722	28	40.2	0.0844
89–92	0.0161	9.0883	22	25.6	0.1273
92–299	0.0019	9.0902	25	28.1	0.1255
299–66	0.1011	9.1913	27	31.2	0.1184
66–219	0.0427	9.2340	25	21.9	0.1425
219–75	0.0717	9.3057	28	22.4	0.1199
75–57	0.0012	9.3069	30	29.2	0.1391

Note that all interevent distances shown are positive and based on relatively large samples (smallest sample size is 11). The weight is used instead of sample size in weighted distance calculation. The standard deviation (last column) applies to the interevent distance; it indicates that in this application most small interevent distances are not significantly different from zero. However, the larger breaks clearly exceed their standard deviations and are probably statistically significant.

In general, relatively many biostratigraphic events originate or terminate at unconformities with or without hiatuses. This explains why the latter generally are characterized by relatively long interevent distances. The events originating or terminating at an unconformity form clusters in the dendrogram because they have relatively small interevent distances between them.

The example used in this paper is for a 7/4 run indicating that, except for the unique events, all events considered occur in seven or more wells, and pairs of events in four or more wells. As shown in Table 5, the estimated interevent distances are based on samples ranging in size (n^* individual estimates, see before in this section) from 11 to 33. In other data sets with fewer wells than the present one, or if the threshold parameters are made smaller than seven and four in the present example, n^* can become very small. Similar considerations apply to the weight W shown in the next column in Table 5. This weight can be regarded as a sample size corrected for variability in the precision of the crossover frequen-

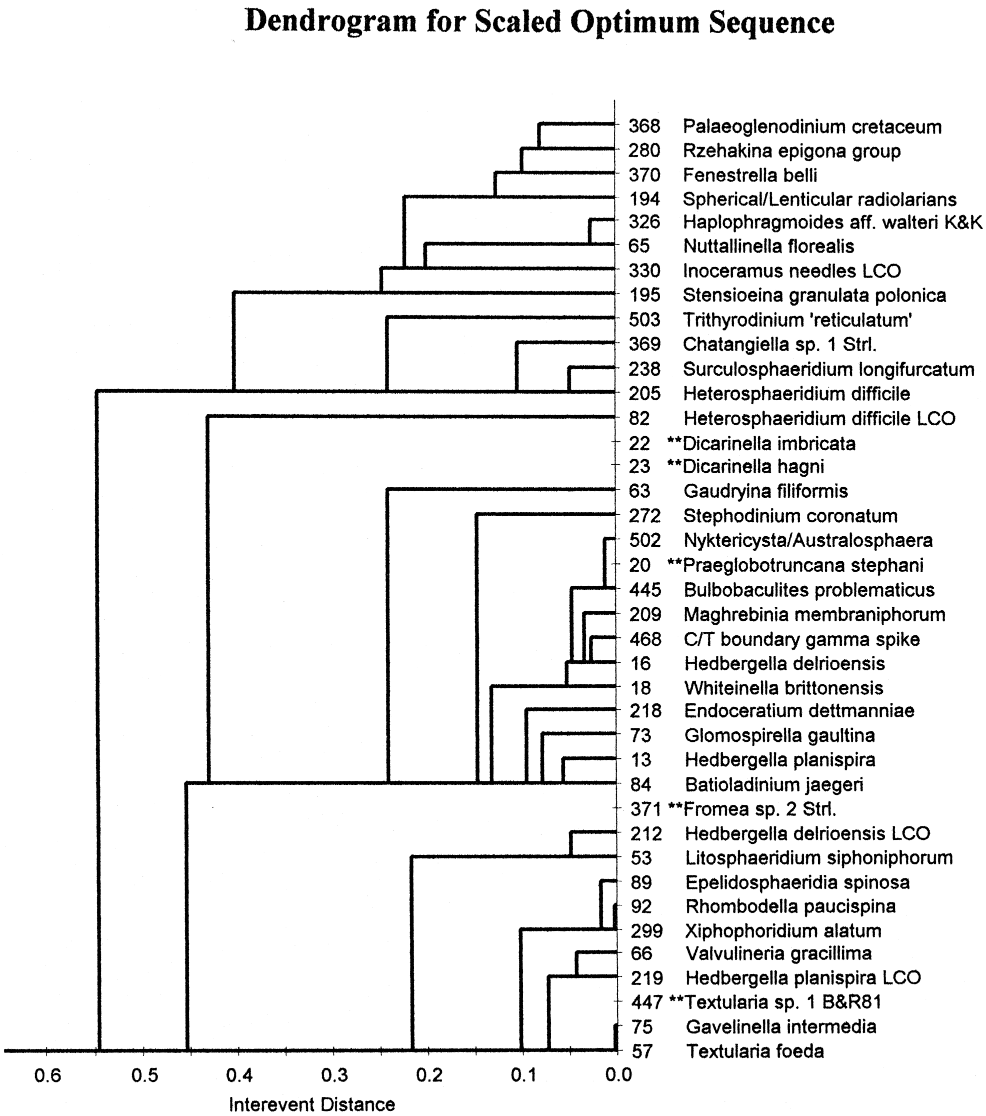

Fig. 4. Dendrogram for scaled optimum sequence corresponding to Table 5. Because of minor reordering, the sequence of events differs from optimum sequence of Fig. 3. Interevent distances involving unique events (marked by double asterisks) are not plotted because estimates of them are relatively imprecise. This diagram represents central part of Fig. 5 in Gradstein et al. (1999).

cies which become more precise when they are based on a larger number of superpositional relations. Interevent distances that are based on very small samples are set equal to zero in the RASC computer program. The rules used are (1) n^* should exceed $10/m$ where m is the minimum number of pairs of events co-occurring in the same well, and (2) W should be greater than 3.5. Both conditions are met for the present example with m equal to 4 so that $10/m = 2.5$. The minimum sample size ($= 11$) exceeds 2.5, and the minimum weight ($= 10.9$) exceeds 3.5. Because n^* and m are both integers, n^* should be at least 4 when m is equal to 3 (a frequently used threshold value), at least 5 when $m = 2$, at least 10 when $m = 1$, at least 3 when $m = 4$, etc. The second rule (2) sets a lower limit of

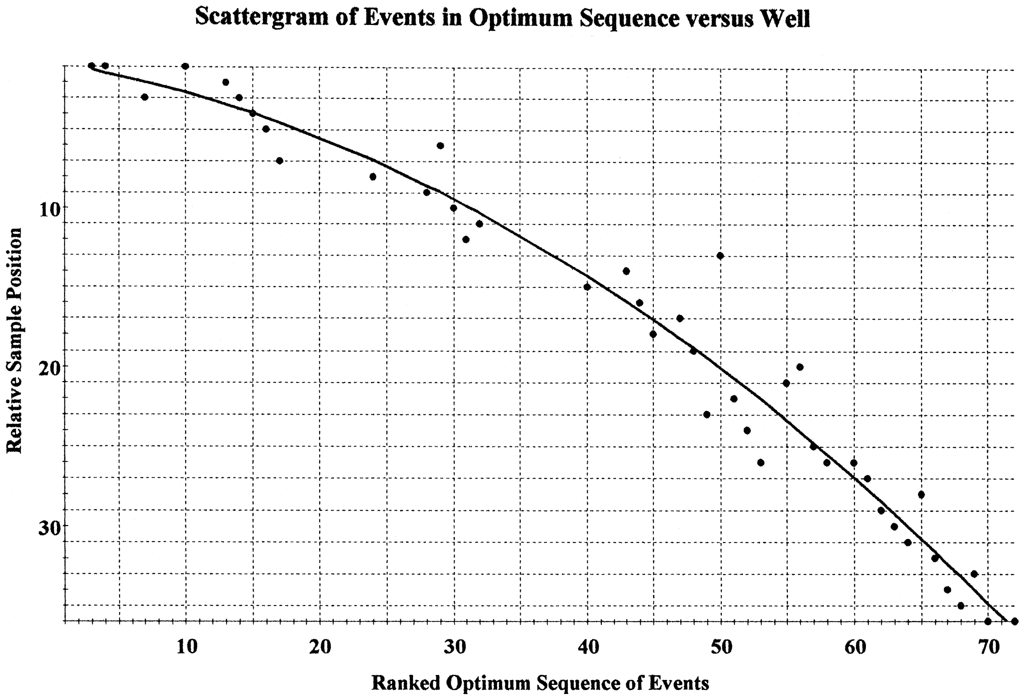

Fig. 5. Line of correlation fitted by least squares method to observed fossil events in Well # 20 (Saga 35/3–4). Ranked optimum sequence positions (Fig. 3) are plotted in horizontal position. Unique events are not included. Relative sample position increases in the stratigraphically downward direction. Optimum sequence events in well samples deviate positively or negatively from line of correlation. Deviations for same event in different wells are shown graphically in Fig. 6.

3.5 on W. Usually, this rule has little effect. It serves as an extra safeguard to prevent the use of relatively imprecise crossover frequencies.

4.2. Marker horizon option— to include events positioned without uncertainty in wells

The use of the probit information in distance calculation implies that it is tacitly assumed that all events considered are subject to the same type and amount of biostratigraphic uncertainty. As already shown in the last part of Fig. 3, this is not a correct assumption because different events may have different standard deviations. The marker horizon option allows the user of the RASC computer program to select a number of events to which zero standard deviations are assigned during the scaling calculation.

In practical applications, logmarkers and, to a lesser degree, seismic events could be examples of events with locations that are known with certainty in all sections containing these events. Such events can be selected as marker horizons. In order to avoid computational difficulties which would arise from adjacent zero-variance events in a scaled optimum sequence, the total number of marker horizons should remain small and marker horizons should be separated by one or more other events in the scaled optimum sequence.

During the early stages of RASC development it soon became clear that the final scaled optimum sequence is hardly changed depending on whether or not a number of events are declared to be marker horizons. Thus the marker horizon option has turned out to have limited practical value. The explanation of this is as follows.

Interevent distances are primarily estimated by the averaging of indirect distances. At most a single direct distance is used (see before). Theoretically, each probit transformation is equivalent to addition of two random variables representing uncertainties in the positions of two stratigraphic events. Suppose that these random variables have unequal variances. Relatively, differences between variances are reduced when random variables are added. Further mixing of such random variables occurs when z-values are differenced for indirect distance calculation.

During this process the frequency distributions of the original random variables become masked. Consequently, the variances of indirect distance estimates are closer in value than the variances of the original random variables used for locating the stratigraphic events along the relative time scale. Even more masking occurs when n^* separate distance estimates are averaged to obtain the final interevent distance estimates. At the end of the process, the possible effect of unequal variances of events has been greatly reduced or eliminated.

The limited practical value of the marker horizon option has a theoretical foundation in the central limit theorem of mathematical statistics. This theorem states that the frequency distribution of the sum of independent random variables converges to normal (Gaussian) form. This convergence can be rapid. For example, if two random variables each have the same uniform distribution, their sum has a triangular distribution which, in shape, is much closer to the Gaussian bell-shaped curve than the original, flat-topped uniform distribution. Likewise, the combination of a Gaussian random variable with a zero-variance marker horizon yields another Gaussian variable, in shape different from the spike associated with zero variance (cf. Agterberg, 1990).

It is noted that, because of similar considerations, it is not advantageous to apply the computationally involved 'modified RASC' method described in D'Iorio and Agterberg (1989) and Agterberg (1990). In this procedure, standard deviations of events comparable to those shown in the last column of Fig. 3 are used in scaling; new lines of correlation are fitted using the resulting scaled optimum sequence, and this iterative process is continued until a final solution is obtained. In practice, a stable end product could not be obtained in some applications. If it could be obtained, the final solution was hardly different from the initial scaling result. For these reasons, this type of iteration is not used at present.

4.3. Treatment of missing and extraneous data.

During scaling, special attention is paid to missing data as well as extraneous data. Pairs of fossils may not co-occur in wells for natural reasons. Additionally, pairs co-occurring in relatively few wells are not considered for statistical averaging because of the second threshold parameter (see before). As a result it can occur frequently that a z-value or the difference between two z-values (used for indirect distance estimation) is missing so that the corresponding distance estimate can not be estimated. Extraneous data arise in indirect distance estimation for two successive events (i and j) when the third event (k) occurs above or below i and j in all wells considered. In general, such data can not be used for calculation.

Because most samples are small, it frequently happens for a pair of events that one of the two events is observed to occur above or below the other one in all sections that contain both events. For example, suppose that two events are coeval on the average so that the probability of observing one above the other is 50% in every section. When there are three sections only, there is 75% probability that one event occurs above or below the other one in one or two sections, and there is 25% probability that it occurs above or below the other one in all three sections. The latter probability increases when the two events are not coeval on the average. Rejection of all small samples with crossover frequencies f_{ij} that are equal to 1 or 0 generally would result in significant loss of information. The following two rules in RASC counteract this imbalance.

The first rule is that unity crossover frequency for sample size n is replaced by $(n-0.5)/n$, while zero crossover frequency is replaced by $0.5/n$. For large n, this transformation has little effect on the crossover frequency. However, it permits the application of the probit transformation and subsequent use for distance estimation of a subset of relative crossover frequencies equal to 0 or 1. In general, during indirect distance information, many pairs of z-values are based on one crossover frequency which is less than 1 (or >0) while the other one is 1 (or 0). Such pairs of events are all used for distance calculation.

On the other hand, it may happen that both crossover frequencies (f_{jk} and f_{jk}) to be used for indirect distance estimation are equal to 1 (or 0). The second rule in RASC is that these crossover frequencies will only be used (after transformation) when the third event (k) occurs above (or below) other events of type k with the property that either one or both of the crossover frequencies (f_{jk} and f_{jk}) associated with it are less than one (or > 0).

Table 6
Normality test output for part of Saga 35/3–4

		Cumulative distance	Second-order difference
Reussella szajnochae	29	0.7059	
Rugoglobigerina rugosa	−30	0.0000	0.9902
Brizalina ex gr. *incrassata*	−34	0.2843	0.9959
Hormosina ovulum	70	1.8801	−1.5958
Red colored planktonic forams	137	1.8801	−1.1787
Globigerinelloides volutus	−277	0.3857	3.6506**
Tritaxia dubia	37	2.8576	−2.5406
Agglutinated benthics LCO (U. Cret.)	48	2.7889	2.2590
Spherical/Lenticular radiolarians	194	4.9793	−4.1190**
Trochamminoides spp.	453	3.0506	2.6857
Gyroidinoides beisseli	322	3.8077	0.1893
R. epigona group	280	4.7542	−0.4993
Haplophragmoides aff. *walteri* K and K	326	5.2013	0.0241
S. granulata polonica	195	5.6726	−0.7180
Inoceramus needles LCO	330	5.4259	2.8179*
W. brittonensis	18	7.9971	−2.8859*
S. coronatum	272	7.6824	0.0736
G. filiformis	63	7.4413	0.7452
H. delrioensis	16	7.9455	−0.1500
H. planispira	13	8.2996	−0.7346
C/T boundary gamma spike	468	7.9192	0.8166
B. jaegeri	84	8.3555	0.3996
Valvulineria gracillima	66	9.1913	−0.9388
R. paucispina	92	9.0883	−0.1773
H. delrioensis LCO	212	8.8081	−0.3051
G. gaultina	73	8.2227	1.2188
L. siphoniphorum	53	8.8562	−0.1839
G. intermedia	75	9.3057	−0.6830
E. spinosa	89	9.0722	1.0884
Endoceratium turneri	−170	9.6115	−1.0606
X. alatum	−299	9.0902	0.8093
Ovoidinium verrucosum	90	9.6938	−0.0698
Globigerinelloides bentonensis	241	10.2276	−0.8045
L. conispinum	50	9.9569	0.4369
H. delrioensis FCO	364	10.1231	−0.0250
O. scabrosum	93	10.2643	0.0279
A. grande	52	10.4333	0.8508
Cribrostomoides nonioninoides	86	11.4531	−1.5335
L. arundum	352	10.9394	0.7818
Uvigerammina una	61	11.2076	0.2771
Kleithriasphaeridium corrugatum	115	11.7528	−0.2296
Heslertonia heslertonensis	−106	11.7528	

The second order differences show how well the position of an event and those of its neighbors fit in with their positions in the scaled optimum sequence (cumulative distance, cf. Table 5). Single events with one or two asterisks are out of place with probabilities of 95% and 99%, respectively. Note, however, that if, on average, none of the events were out of place, approximately 1 out of 20 events would still show one (or two) asterisks.

4.4. Normality test—to compare observed relative positions of events with scaled optimum sequence

A useful test developed during the first stage of RASC development (cf. Gradstein, 1984) is the normality test. This test serves three purposes: (1) it is used to compare individual sections with the scaled optimum sequence to find events that are out of place; (2) it provides an overall normality test to test the hypothesis that the probit transformation is suitable for use during scaling; and (3) the autocorrelation of successive interevent distances in the scaled optimum sequence can be studied.

An example of normality test output is shown in Table 6. The first columns show successive relative positions of events in a well. Hyphens in the second column indicate events observed in the same samples as events listed above them in the table. The cumulative RASC distance of each event is also shown. If the order of the events in the section would be according to the scaled optimum sequence, the cumulative RASC distance would display monotonic increase in the stratigraphically downward direction. Events observed to be too high in the section have higher than expected RASC distance; those which are too low have smaller RASC distance than their neighbors.

The second-order differences in the last column were computed by subtracting twice the RASC distance of an event from the sum of the RASC distances of its neighbors. For example, in Table 6 the first three events occur in a single sample. The second event has 0 cumulative value because it is the first event in the scaled optimum sequence. Consequently, its second-order difference is equal to the sum of the values of the other two events in the same sample. When successive events in the well record are not from the same sample, one would expect some increase in value. For this reason, a correction then is applied for the average increase in RASC distance between successive samples in the stratigraphically downward direction. The normality test procedure accentuates the occurrence of anomalously high or low events in the section. Events that occur too high or too low with probabilities of 95% and 99% are marked by one or two asterisks, respectively. The corresponding confidence limits were estimated by fitting a truncated normal distribution to the central 60% of the second-order differences estimated for all sections. It should be kept in mind that the events marked by asterisks are not necessarily out of position. Most differences between order of events in a section and scaled optimum sequence are of random nature. Only exceptionally large discrepancies can be called anomalous. On the average, even when there are no anomalies, 5% and 1% of events in sections receive one and two asterisks, respectively.

The second-order differences from all sections can be subjected to Pearson's χ^2 test for goodness of fit of the normal (Gaussian) distribution (Table 7). If there are anomalous events in the sections, this would be visible in the tails of this overall frequency distribution where the observed frequencies then significantly would exceed the expected frequencies.

Finally, the frequency distribution of the second-order differences provides information on the autocorrelation of the estimated interevent distances of the scaled optimum sequence. Successive interevent distances are not statistically independent because, in

Table 7
Comparison of observed and expected frequencies of occurrence of 734 second-order difference values in all wells (including values of Table 6)

Class No.	Observed	Expected	Difference	Delta
1	99	73.400	25.600	5.073*
2	51	73.400	−22.400	3.884*
3	69	73.400	−4.400	0.150
4	68	73.400	−5.400	0.226
5	76	73.400	2.600	0.052
6	78	73.400	4.600	0.164
7	81	73.400	7.600	0.447
8	57	73.400	−16.400	2.082
9	56	73.400	−17.400	2.343
10	99	73.400	25.600	5.073*
$\chi^2 =$				19.493**

Theoretically, the 734 values would obey a Gaussian frequency distribution. The limits of 10 classes were chosen so that each class has expected frequency of 73.4. The observed frequencies of the smallest values and the largest values exceed 73.4. Each difference can be transformed into a delta value (last column) which is approximately distributed as the χ^2 statistic with a single degree of freedom. One and two asterisks indicate that a difference is statistically significant with probability of 95% and 99%, respectively. The sum of all deltas ($=19.5$) is distributed as χ^2 with approximately seven degrees of freedom. Overall, a slight but statistically significant departure from the Gaussian model is indicated.

part, they were calculated from the same crossover frequencies. For example, as shown in more detail in RASC output, the 734 values in the example of Table 7 are equivalent to only 417 statistically independent values. Similar considerations apply to the original n^* distance estimates from which an interevent distance was computed.

The danger of averaging values that are not statistically independent is that bias in the computed average values may be introduced. In the normality test, this type of bias could be avoided by fitting the frequency distribution to the second-order differences without preconceived knowledge about lack of statistical independence. However, this type of simple correction can not be applied to the standard deviations of the interevent distances (see Table 5) which are likely to be too small because of lack of statistical independence of the original distance estimates. This difficulty can only be resolved by the introduction of computationally laborious methods such as the jackknife (cf. Agterberg, 1990) not included in RASC at present. Fortunately, the interevent distances themselves are not subject to bias of this type.

5. Variance analysis—determination of the standard deviations of fossil events

In general, the number of optimum sequence events per well is less than the total number of

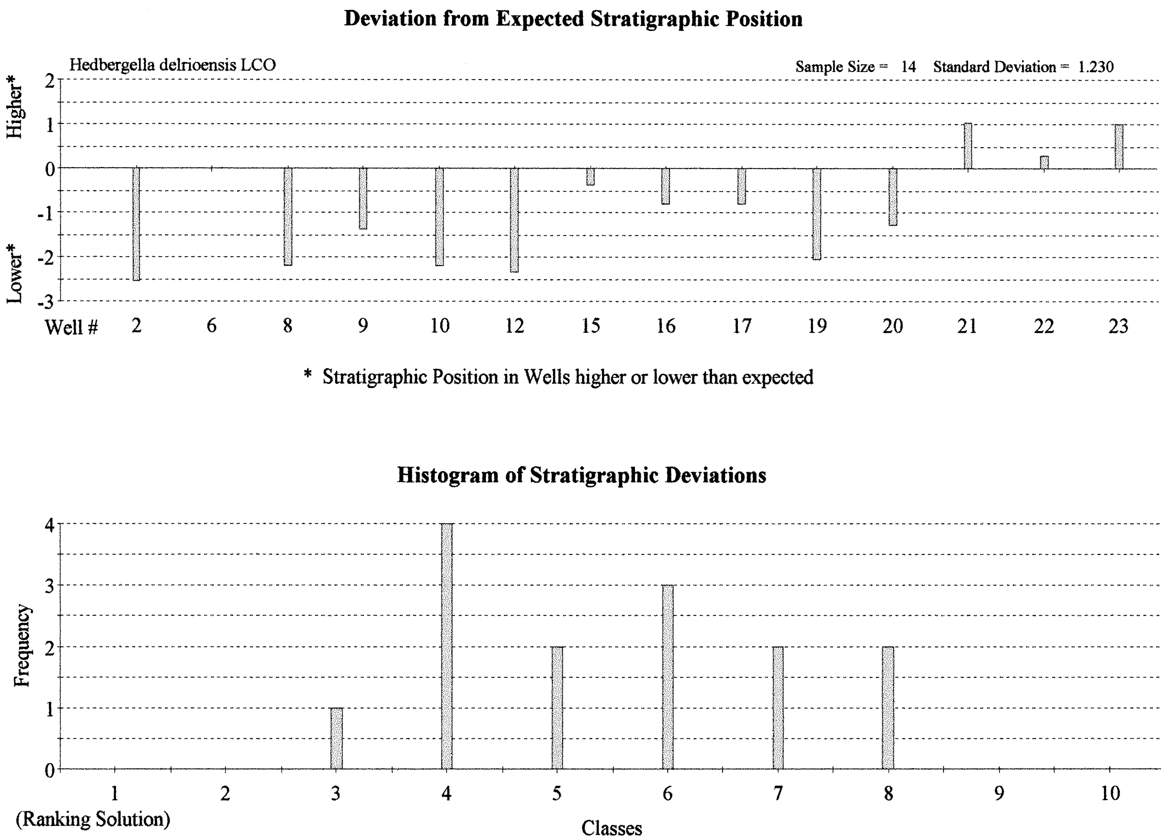

Fig. 6. Upper part: Deviation from expected stratigraphic position per well for *H. delrioensis* LCO (Last Common Occurrence). Well order from north (left) to south (right). Deviations from lines of correlation including line shown in Fig. 6. Because the number of negative deviations is relatively large, it is possible that position of this event should be slightly lower in optimum sequence (cf. Agterberg et al., 1998). This is a relatively good marker as indicated by its (below 'average') standard deviation. Lower part: Histogram of deviations (see text for explanation of class limits).

events in the optimum sequence. For each section, these events can be plotted against their optimum sequence positions in a scattergram. In general, the cluster of points is curved either upwards or downwards. A curve (line of correlation) can be fitted by least squares. In RASC a second-degree polynomial curve with three coefficients is used. An example is shown in Fig. 5 using Saga 35/3–4 (Well # 20). Precautions have been taken that the best-fitting curve does not show a maximum or minimum over the optimum sequence range to prevent violation of the law of superposition of strata. In practice, problems of this type have been observed to occur only rarely. The curve-fitting can be performed with either optimum sequence (ranking) or scaled optimum sequence position as the independent (explanatory) variable.

A simple test to determine possible outliers that occur stratigraphically too high or too low in a well consists of flagging those events which are more than two (residual) standard deviations removed from the line of correlation. This test is available in RASC (Agterberg and Gradstein, 1996).

Each point in Fig. 5 represents an optimum sequence event. Its deviation from the line of correlation can be measured in the vertical direction. Events on or near the line of correlation have small deviations. If an event has small deviation in all wells where it occurs, it is likely to be a good marker. This aspect can be quantified by computing the standard deviation of all events. For the example, these standard deviations were already shown previously in Fig. 3 in comparison with 'average' standard deviation calculated from all data in all wells as follows. The variance of all deviations is computed for each well and the resulting variances are averaged. The 'average' standard deviation is the square root of this average variance. Individual well variances are provided in RASC output. High quality wells are characterized by relatively small well variance.

Fig. 6 provides an example of variance analysis for a single event (*H. delrioensis* LCO). Its deviations are either positive or negative, where positive deviations indicate wells where the event occurs stratigraphically higher than expected, and negative the opposite. The average deviation is slightly negative suggesting that it may be useful to adjust the position of this event in the optimum sequence. (For a possible explanation of the sign of average deviation and adjustment, see Agterberg et al., 1998.) The wells are arranged according to latitude but there is no obvious geographic variation pattern in the deviations of Fig. 6 suggesting absence of diachronism of the event. There is no significant positive or negative skewness. The standard deviation of this event is less than the average standard deviation based on all wells containing the event (cf. Fig. 3). This event is a relatively good marker (also see Gradstein and Agterberg, 1998).

5.1. *Histogram of deviations from lines of correlation*

The bottom part of Fig. 6 is a histogram of the deviations. The histogram is centered about the mean deviation. The class limits are determined by the 'average' standard deviation (Well SD) in order to provide a common scale for all events. Each class width is half of this unit. Together, the 10 classes are five units (Well or 'ave' SD) wide. Deviations less than -2.5 or greater than 2.5 times the unit are added to first and last class, respectively. A good marker event shows a relatively sharp peak in the

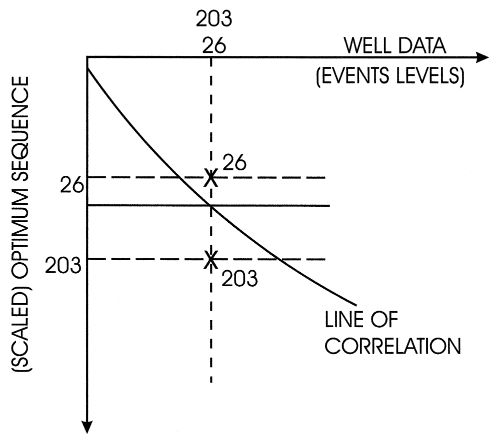

Fig. 7. Schematic diagram to illustrate projection of hypothetical deviations (cf. Figs. 5 and 6) for two fossil events numbered 26 and 203 observed to be coeval in a well. Horizontal axis is for relative sample position; vertical axis gives position of events in (scaled) optimum sequence which is the same for all wells. Arrows along axes point stratigraphically downward. This type of projection compensates for differences of curvature in the lines of correlation.

middle of the histogram, and a poor marker can occur in all 10 classes, perhaps with peaks in its first and last classes.

Variance analysis has several advantages: (1) It allows identification of good markers. These are useful for stratigraphic correlation because lines of correlation between wells based on fossil events with small standard deviations show few if any crossovers; on the other hand, sequences of events with large standard deviations may be indicative of specific types of environment (e.g., sand reservoirs deposited by turbidity currents; cf. Gradstein and Agterberg, 1998). (2) It is possible to discern diachronous biostratigraphic events. (3) The frequency distribution of the deviations can be checked for positive or negative skewness which may be related to other factors including reworking and rate of evolution or extinction of a taxon. (4) The overall quality of a well can be assessed.

The possible importance of diachronism was emphasized by Drooger (1974) who also discussed the fact that occurrence of fossils may be restricted to geographically delineated domains. RASC has limited capabilities of dealing with paleoecology. However, we are supplementing RASC with techniques specifically dealing with paleogeographical distribution patterns (COR and TRACE, see Sections 7 and 7.1).

5.2. *Event ranges—to estimate uncertainty of event positions along relative time scale*

The position of an event in the (scaled) optimum sequence is an average position. As shown in the preceding section, individual events in wells can deviate from their average position and occur stratigraphically higher or lower than expected. For each event, the maximum and minimum deviation can be

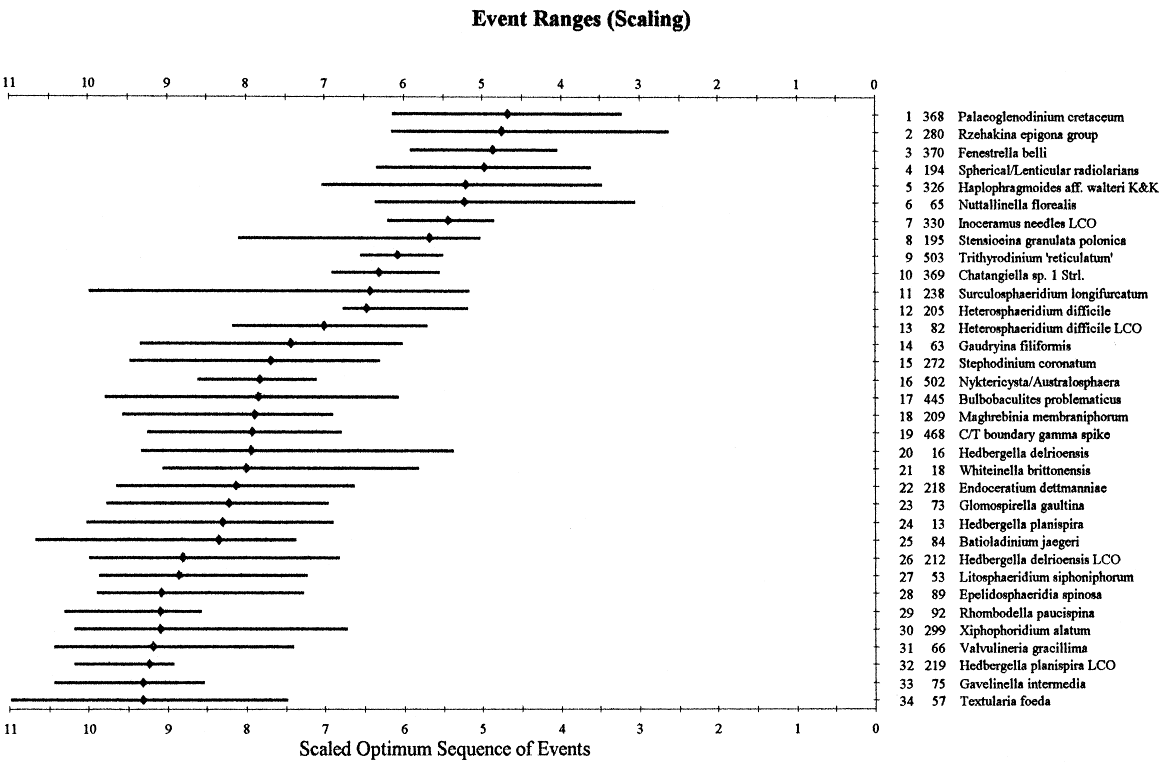

Fig. 8. Projection method of Fig. 7 used to construct event ranges for scaled optimum sequence of Fig. 4. Each event range shows as a horizontal line, together with the event's cumulative RASC distance. Arrow of time points upwards and to the left.

determined. These are the endpoints of an event range. However, the lines of correlation are commonly curved with different curvature in different wells. Fig. 7 shows the method used to project fossil events from all wells onto a single, common scale. Every observed event is projected onto the line of correlation along a line which is parallel to the (scaled) optimum sequence. Fig. 8 shows the resulting event ranges for the scaled optimum sequence of Fig. 4.

Another method can be used to construct event ranges. For every event in the (scaled) optimum sequence we can determine all events showing at least one inconsistency with it. These events form a range bounded by the stratigraphically lowest and highest events with which it is locally inconsistent. Fig. 9 shows this type of event crossover range for the optimum sequence example of Fig. 3. If first and last occurrences are known for a fossil taxon, a conservative estimate of its total range would extend from the bottom of the range of its first occurrence to the top of the range of its last occurrence.

5.3. RASC summary table

At the end of a RASC run, a summary table (see Table 8 for the example) is produced with selected statistics. The user may wish to inspect this table early on in order to obtain a good idea of the overall quality of the result. For example, a large number of cycles or a large number of events with six or more penalty points in one or more wells are statistics suggesting poor quality of the result. The user may then decide to change threshold parameters and rerun the data set, or to initiate detailed editing to deter-

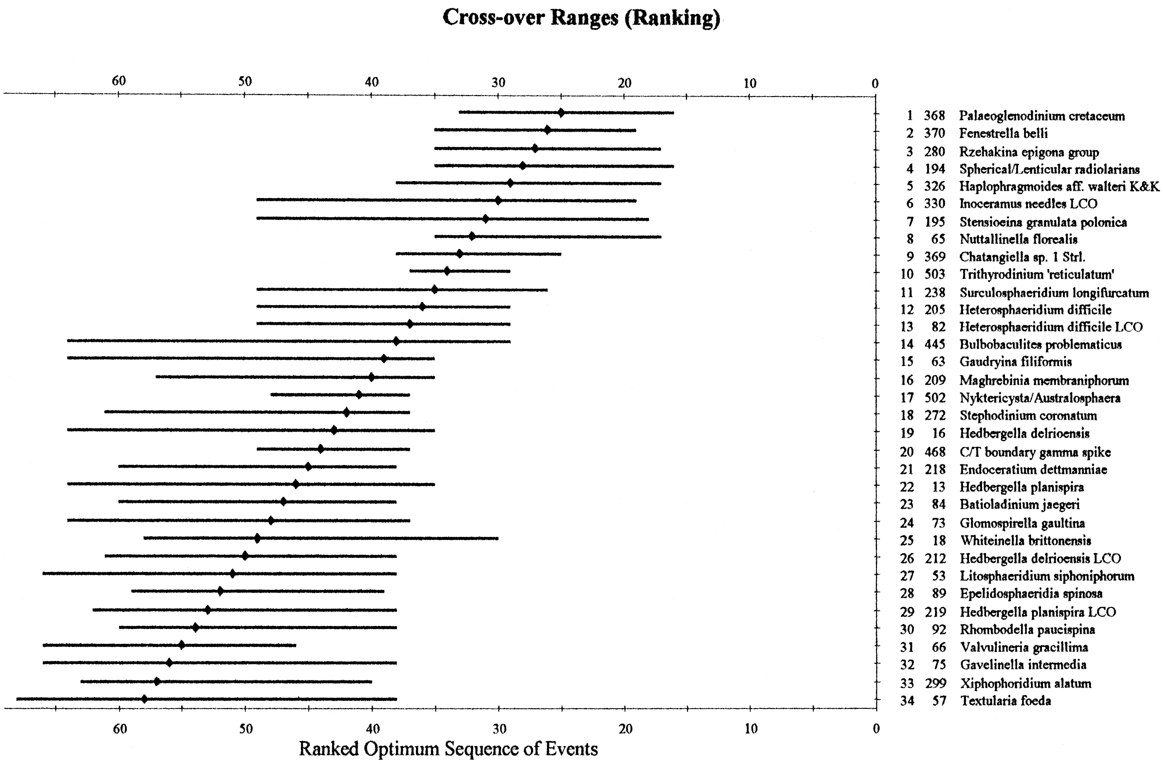

Fig. 9. Crossover ranges for ranked optimum sequence of Fig. 3. Each event range extends to the stratigraphically highest and lowest event in the optimum sequence with which it shows inconsistency. Arrow of time points upwards and to the left.

Table 8
RASC summary table for 7/4 run

Number of names (taxa) in the dictionary	592
Number of wells	31
Number of dictionary taxa in the wells	517
Number of event records in the wells	1753
Number of cycles prior to ranking	21
Number of events in the optimum sequence	72
Number of events in optimum sequence with SD < ave SD	43
Number of events in the final scaled optimum sequence (including unique events shown with **)	81
Number of stepmodel events with more than six penalty points after scaling	14
Number of normality test events shown with * or **	78
Number of AAAA events in scaling scattergrams	31

These statistics provide quick insight into the size and quality of the data set. Most entries have been discussed in detail in the body of the text. The last entry (number of AAAA events in scaling scattergrams) refers to a graphical output showing well data vs. scaled optimum sequence. Each AAAA event is more than two (residual) standard deviations removed from the line of correlation for the well in which it occurs.

mine which events are out of place in which sections.

6. CASC—correlation and standard error calculation

The preceding pages dealt with zonation of fossil events and tests to discern ‘good and bad’ events, out of place events, and ‘good and bad’ wells. Below we will discuss correlation of RASC events and standard error calculation, grouped as a technique called CASC, for *C*orrelation *a*nd *S*tandard Error *C*alculation. The technique is an extension of so-called ‘graphic correlation’, as discussed in more detail in Gradstein et al. (1985). Fig. 12 in Gradstein et al. (1999) provides an example of CASC correlation with error bars.

Fig. 10 illustrates how the (scaled) optimum sequence can be used for correlation between wells. Any event in the (scaled) optimum sequence in a well can be projected onto the line of correlation for this well by constructing a line parallel to the well axis for relative depth (horizontal in Fig. 10). Projection of the point of intersection onto this relative depth axis yields the probable position of the event considered. Probable positions can be calculated for events not observed to occur in the well. Additionally, the standard deviation of the calculated value on the line of correlation can be estimated. For this, the standard deviation of the event considered is multiplied by a geometrical factor determined by the position of the event in the (scaled) optimum sequence relative to the positions of all events present in the well. The equation of this geometrical factor is as used in mathematical statistics to calculate the error of estimated values in regression analysis (cf. Agterberg and Gradstein, 1997b).

Multiplication of this standard deviation by 2 yields a symmetrical, approximate 95% confidence interval along the relative depth axis which, together with the probable position of the event, can be projected onto the depth axis for the well. This final projection is performed by determination of the intersection points of the three vertical lines per event with the so-called line of observation (top part of Fig. 10). The line of observation is a cubic interpolation spline constructed through the depths of all

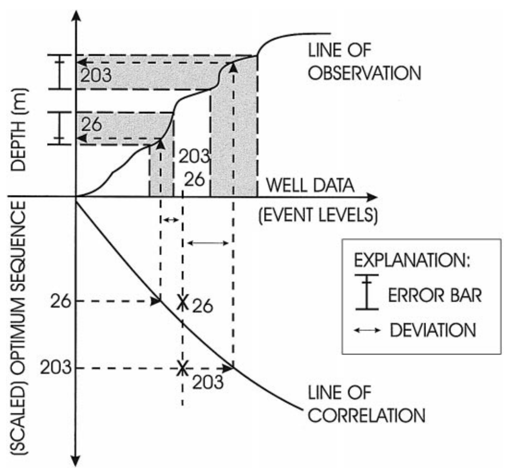

Fig. 10. Schematic diagram to illustrate construction of CASC probable event depths with error bars (95% confidence intervals). Each event (26 or 203, cf. Fig. 7) is horizontally projected onto the line of correlation to obtain its probable position along the well data axis. Its symmetrical error bar along this axis is projected onto the line of observation to obtain asymmetrical error bar along the depth scale for the well. See text for further explanation.

samples in the well which contain one or more (scaled) optimum sequence events.

Projection of the symmetrical error bar (95% confidence interval) along the relative depth axis generally results in an asymmetrical error bar along the true depth axis. Of course, the exact depth of the probable positions of events belonging to the (scaled) optimum sequence is not known. However, the line of observation generally is robust, in that it is not strongly dependent on positions of single samples in the well, and can be used for this type of projection.

Probable depths are not shown in CASC output if uncertainty in position as indicated by the error bar in the well exceeds 200 m. Comparison between observed and calculated depths of events in wells is performed as follows. Observed values and estimated (probable) values considered until now share the same expected values. However, uncertainties associated with observed values usually are much greater than those of the estimated (probable) values. Standard deviations of observed values can be computed by multiplying the event standard deviation by a geometrical factor for individual observations used during regression analysis. Multiplication by two yields wider error bars (approximately 95% confidence intervals). If an observed value plots outside its 95% confidence interval, there is the possibility that it occurs either stratigraphically too high or too low in the well considered. This type of comparison between observed and probable values constitutes another normality test.

In CASC graphics it is possible to select a specific event for flattening. This means that in the display the wells are moved up or down in such a way that the lines of correlation of a selected event becomes horizontal.

7. Correspondence analysis—clustering of fossil events within RASC zones according to well location

The optimum sequences obtained by ranking and scaling are based on the assumption that all events can be projected onto a single scale directed according to the arrow of time. This projection is carried out irrespective of the (paleo)geographic locations of the wells. By means of variance analysis it is possible to study the validity of this assumption; e.g., events can be checked for the presence or absence of diachronism after ordering the wells according to their geographic position (latitude). There are other methods by which geographic location of the wells can be considered including correspondence analysis.

Bonham-Carter et al. (1986) proposed the use of correspondence analysis in a study dealing with Cenozoic Foraminifera from 36 offshore wells along the northwestern Atlantic Margin. The wells were ordered from north to south so that well numbers could be used for location (latitude). The other variable considered was presence (= 1) or absence (= 0) of the fossil events (last occurrences of taxa) in the wells.

Like Bonham-Carter et al., we have used the algorithm described by Hill (1979) for correspondence analysis by means of reciprocal averaging,

Table 9
First part (well scores) of correspondence analysis output for interval zone within optimum sequence for 7/4 run extending from event # 368 through 205

	Well #	Well score
1	27	0.000
2	26	25.314
3	4	34.909
4	20	36.699
5	12	49.102
6	22	52.079
7	1	53.306
8	25	54.965
9	16	56.474
10	15	57.490
11	14	60.888
12	30	63.208
13	28	63.254
14	17	70.268
15	11	74.803
16	13	74.818
17	10	80.899
18	9	81.184
19	8	85.066
20	6	91.030
21	3	91.628
22	7	91.753
23	29	100.000

Events occurring in fewer than five wells were not considered. Well scores of first eigenvector are normalized from 0 to 100. Since original order of wells (second column) runs from north to south, only a vague tendency is indicated that most southern wells occur in top part and most northern wells occur in bottom part of this table.

because it can deal with very large data sets. Bonham-Carter et al. pointed out that this technique produces results similar to those obtained by means of seriation (Burroughs and Brower, 1982). The first eigenvector resulting from correspondence analysis of presence–absence data from ordered sample locations tends to diagonalize the presences. Both the taxa and the wells receive scores according to which they are ordered along the axis of time as well as in the north–south direction. Bonham-Carter et al. concluded that useful results were obtained by restricting the analysis to taxa from specific RASC zones, rather then including all taxa. Severe distortions of stratigraphic order occurred when events from the entire Cenozoic were used simultaneously.

Within RASC zones (time-slices), it could be seen from the scores that the occurrence of some planktonic taxa was restricted to the southern part of the study area whereas some benthonic taxa occurred in the north only. We have included this method in the computer program COR with the following additional features.

For any slice from a (scaled) optimum sequence defined by the events at its top and bottom, correspondence analysis can be performed on events that occur in at least a minimum number of wells. If this threshold is set equal to 1, all events will be used. However, small-sample effects, like events occurring in a single well necessarily ending up on the diagonal, can be avoided by choosing a threshold greater than 1. COR also contains the optional method of simultaneously ordering the event according to their average values along the time axis as well as the north–south direction. Finally, it can provide various types of constrained solutions. For example, events can be clustered along the diagonal while their order in time as given in the (scaled) optimum sequence is preserved.

Tables 9 and 10 show correspondence analysis results obtained by COR for the example using the RASC interval extending from event 368 (*P. cretaceum*) to 205 (*H. difficile*). Table 9 shows the well scores, and Table 10 the event scores. The last part of Table 10 illustrates degree of diagonalization achieved. The first eigenvalue is equal to 0.266 which is considerably less than the eigenvalues reported by Bonham-Carter et al. (1986) indicating that north–south differentiation of taxa, if it exists at all, is much less pronounced in the current data set (Tables 9 and 10).

7.1. Traceability—estimation of probabilities of occurrence of taxa in wells

Agterberg and Gradstein (1999) describe the use of logistic regression as a method to study traceabil-

Table 10
Second part (event scores) of correspondence analysis output

No.	Score	Well number																						
		27	26	4	20	12	22	1	25	16	15	14	30	28	17	11	13	10	9	8	6	3	7	29
65	44.564	X		X				X		X	X	X												
194	48.435	X	X	X	X	X	X		X	X	X	X	X	X				X						
195	49.284	X	X		X	X	X	X		X	X	X	X		X		X							
326	55.261			X	X	X				X	X	X							X	X				
280	57.693				X	X	X				X	X				X	X		X					
238	60.116		X	X		X		X	X	X	X	X	X			X		X	X	X	X		X	
330	64.041				X	X		X		X	X	X	X	X			X	X	X	X	X	X	X	
370	64.507			X				X		X	X	X	X			X	X	X	X	X		X	X	
369	67.500						X		X	X	X	X			X		X	X	X	X	X		X	X
205	67.550					X	X	X		X		X	X				X	X	X	X	X	X	X	X
368	68.667									X	X	X	X	X	X	X	X	X	X	X		X	X	X
503	71.138										X	X						X	X	X	X		X	

The matrix of presences/absences on the right-hand side shows degree of diagonalization achieved. Events near the top would tend to occur in the south, those in the middle may occur evenly throughout the entire study area, and those near the bottom toward the north. However, these trends are weak and may not be meaningful as indicated by the relatively small first eigenvalue (= 0.266). Negative results of this type favor validity of the RASC approach which performs best when all events are evenly distributed throughout the study region.

ity of taxa. This approach shows points of similarity with correspondence analysis. It works on single taxa only. The geographic coordinates of the samples (wells) form part of the input and are used to create independent (explanatory) variables for logistic regression. The presence–absence data for the taxon provide the values of the dependent variable. The result is a set of probabilities of occurrence for the taxon.

Typically, the output from the computer program TRACE (for traceability) consists of a plateau with relatively high probabilities of occurrence of the taxon possibly bounded at one side by a relatively sharp transition zone toward zero (or negligibly small) probabilities. The average probability of occurrence on the plateau is roughly equal to the proportion of wells on this plateau that contain the taxon. Often it can be assumed that the taxon existed on the plateau only. For examples of applications of TRACE, see Agterberg and Gradstein (1999).

8. Concluding remarks

The RASC method provides a rapid means of analyzing large biostratigraphic data sets which are subject to inconsistencies involving two or more fossil events. Other types of stratigraphic events can be integrated with the biostratigraphic data like log-markers or seismic horizons. It should be kept in mind that the type of mathematical analysis performed is only possible once a comprehensive biostratigraphic data set is available. The user, in general, spends far more time on data collection and preparing the data for analysis than on the mathematical applications themselves. The data entry aspect is being facilitated by incorporating a Makedat program into RASC by means of which taxa, events and wells can be readily selected and edited before application of the techniques described in this paper.

Analysis of RASC and CASC results has become easier with the development of RASC for Windows. Possible improvements of RASC for the future include better explanations of results during execution of the computer program and incorporation of guidelines to help the user decide on further action (e.g., revision of threshold parameters) required to obtain better results.

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References

Agterberg, F.P., 1990. Automated Stratigraphic Correlation. Elsevier, Amsterdam, 424 pp.

Agterberg, F.P., Byron, D.N., 1990. FORTRAN 77 microcomputer programs for ranking, scaling and regional correlation of stratigraphic events. In: Agterberg, F.P., Bonham-Carter, G.F. (Eds.), Statistical Applications in the Earth Sciences, Geol. Surv. Can. Paper 89-9.

Agterberg, F.P., Gradstein, F.M., 1988. Recent developments in quantitative stratigraphy. Earth-Science Reviews 25, 1–73.

Agterberg, F.P., Gradstein, F.M., 1994. Three-dimensional ranking of biostratigraphic events. In: Papers and Extended Abstracts 1st Annual Conference, Int. Assoc. Math. Geol., Mont Tremblant, Canada, October 1994.

Agterberg, F.P., Gradstein, F.M., 1996. RASC and CASC: Biostratigraphic Zonation and Correlation Software, version 15. F. Agterberg, 490 Hillcrest Avenue, Ottawa, Canada K2A 2M7.

Agterberg, F.P., Gradstein, F.M., 1997. Measuring the relative importance of fossil events in quantitative stratigraphy. In: Pawlowsky-Glahn, V. (Ed.), Proceedings IAMG'97. CIMNE, Barcelona, Part 1, pp. 349–354.

Agterberg, F.P., Gradstein, F.M., 1997. Sequencing, scaling and correlation of stratigraphic events. In: Naiwen, W., Remane, J. (Eds.), Proceedings 30th Int. Geol. Congr., Vol. 11, Beijing, China, VCP, Zeist, pp. 29–37.

Agterberg, F.P., Gradstein, F.M., 1999. Scaling and the palaeogeographical distribution of stratigraphic events. In: Harff, J. et al. (Eds.), Proceedings International Symposium on Computerized Modeling of Sedimentary Systems, held in Guestrow, October 1996. Springer, Berlin, pp. 263–273.

Agterberg, F.P., Nel, L.D., 1982a. Algorithms for the ranking of stratigraphic events. Computers and Geosciences 8, 69–90.

Agterberg, F.P., Nel, L.D., 1982b. Algorithms for the scaling of stratigraphic events. Computers and Geosciences 8, 163–189.

Agterberg, F.P., Oliver, J., Lew, S.N., Gradstein, F.M., Williamson, M.A., 1985. CASC Fortran IV interactive computer program for correlation and scaling in time of biostratigraphic events. Geological Survey of Canada, Open File Report 1179.

Agterberg, F.P., Gradstein, F.M., Nel, L.D., Lew, S.N., Heller, M., Gradstein, W.S., D'Iorio, M.A., Gillis, D., Huang, Z.,

1989. Program RASC (Ranking and Scaling) version 12. Comm. Quantitative Stratigraphy. Bedford Institute Oceanography, Dartmouth, N.S., Canada.

Agterberg, F.P., Gradstein, F.M., Cheng, Q., 1998. Stratigraphic correlation on the basis of fossil events. In: Buccianti, A. et al. (Eds.), Proceedings 4th Annual Confer. Int. Assoc. Math. Geol., Ischia, October 1998, pp. 743–748.

Blank, R.G., Ellis, C.H., 1982. The probable range concept applied to the biostratigraphy of marine microfossils. J. Geol. 90, 415–433.

Bonham-Carter, G.F., Gradstein, F.M., D'Iorio, M.A., 1986. Distribution of Cenozoic Foraminifera from the Northwestern Atlantic Margin analyzed by correspondence analysis. Computers and Geosciences 12, 621–635.

Burroughs, W.A., Brower, J.C., 1982. SER, a FORTRAN program for the seriation of biostratigraphic data. Computers and Geosciences 8, 137–148.

D'Iorio, M.A., Agterberg, F.P., 1989. Marker event identification technique and correlation of Cenozoic biozones on the Labrador Shelf and Grand Banks . Bull. Can. Petroleum Geol. 37, 346–357.

Doeven, P.H., Gradstein, F.M., Jackson, A., Agterberg, F.P., Nel, L.D., 1982. A quantitative nannofossil range chart. Micropal. 28, 85–92.

Drooger, C.W., 1974. The boundaries and limits of stratigraphy. Proc. Kon. Ned. Akad. Wet. Ser. 11B 17, 159–176.

Finney, D.J., 1971. Probit Analysis, 3rd Edn. Cambridge Univ. Press, 333 pp.

Gil-Bescos, E., Egozcue, J.I., Pawlowsky-Glahn, V., Agterberg, F.P., 1998. An extreme-value approach to scaling biostratigraphic events. In: Proceedings 4th Annual Confer. Int. Assoc. Math. Geol., Ischia, October 1998.

Gradstein, F.M., 1984. On stratigraphic normality. Computers and Geosciences 19, 43–57.

Gradstein, F.M., Agterberg, F.P., 1982. Models of Cenozoic foraminiferal stratigraphy—northwestern Atlantic margin. In: Cubitt, J.M., Reyment, R.A. (Eds.), Quantitative Stratigraphic Correlation. Wiley, Chichester, pp. 119–173.

Gradstein, F.M. and Agterberg, F.P., 1998. Uncertainty in stratigraphic correlation. In: Gradstein, F.M. et al. (Eds.), Sequence Stratigraphy—Concepts and Applications Elsevier, Amsterdam, pp. 9–29.

Gradstein, F.M., Bäckström, S.A., 1996. Cainozoic biostratigraphy and palaeobathymetry, northern North Sea and Haltenbanken. Norsk Geologisk Tidsskrift 76, 3–32.

Gradstein, F.M., Agterberg, F.P., Brower, J.C., Schwarzacher, W., 1985. Quantitative Stratigraphy. Reidel, Dordrecht and UNESCO, Paris, 598 pp.

Gradstein, F.M., Huang, Z., Merrett, D., Ogg, J.G., 1992. Probabilistic zonation of Early Cretaceous microfossil sequences, Atlantic and Indian Oceans, with special reference to ODP Leg 123. In: Proceedings Ocean Drilling Project, Scientific Results, Leg 123, pp. 759–777.

Gradstein, F.M., Kaminski, M.A., Agterberg, F.P., 1999. Biostratigraphy and paleoceanography of the Cretaceous Seaway between Norway and Greenland. Earth-Science Reviews 46, 27–98.

Gradstein, F.M., Kaminski, M.A., Berggren, W.A., Kristiansen, I. L, D'Iorio, M.A., 1994. Cainozoic biostratigraphy of the North Sea and Labrador Shelf. Micropal. 40 (Supplement 1994), 1–152.

Harper, C.W. Jr., 1984. A Fortran IV program for comparing ranking algorithms in quantitative biostratigraphy. Computers and Geosciences 10, 3–29.

Hay, W.W., 1972. Probabilistic stratigraphy. Eclogae Geol. Helv. 65, 255–266.

Heller, M., Gradstein, W.S., Gradstein, F.M., Agterberg, F.P., Lew, S.N., 1985. RASC Fortran 77 computer program for ranking and scaling of biostratigraphic events. Geological Survey of Canada, Open File 1203.

Hill, M.O., 1979. DECORANA—a FORTRAN program for detrended correspondence analysis and reciprocal averaging: ecology and systematics. Cornell University, Ithaca, New York, 52 pp.

Kendall, M.G., 1975. Rank Correlation Methods. Griffin, London, 202 pp.

Macellari, C.E., 1986. Late Campanian–Maastrichtian ammonite fauna from Seymour Island (Antarctic Peninsula). J. Paleont. 60 (II), 1–55.

Marshall, C.R., 1995. Distinguishing between sudden and gradual extinctions in the fossil record; predicting the position of the Cretaceous–Tertiary iridium anomaly using the ammonite fossil record on Seymour Island, Antarctica. Geology 23, 731–734.

Schioeler, P., Wilson, G.J., 1993. Maastrichtian dinoflagellate zonation in the Dan Field, Danish North Sea. Review of Palaeobotany and Palynology 78, 321–351.

Strauss, D., Sadler, P.M., 1989. Classical confidence intervals and Bayesian probability estimation for ends and local taxon ranges. Math. Geol. 21, 411–427.

Van Buggenum, J.M., 1991. The range chart method: an approach toward the creation of a range chart for biostratigraphic events. Geologie Africaine, Coll. Geol. Libreville, Recueil des Communic. 6–8 May 1991, pp. 181–187.

Whang, P., Zhou Di, 1997. The application of RASC/CASC methods to quantitative biostratigraphic correlation of Neogene in northern South China Sea. In: Naiwen, W., Remane, J. (Eds.), Proceedings 30th Int. Geol. Congr., Vol. 11., Beijing, China. VSP, Zeist.

Williamson, M.A., 1987. Quantitative biozonation of the Late Jurassic and Early Cretaceous of the East Newfoundland Basin. Micropal. 33, 37–65.